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ROTATIONAL EFFECTS ON THE DYNAMICAL INSTABILITY OF LUMINOUS BLUE VARIABLES

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ABSTRACT

Rotation influences the dynamical stability of a star in both direct and indirect ways. Directly, it supplies rotational kinetic energy to the star and changes the star's hydrostatic structure. Indirectly, it influences the possible course of stellar evolution. Calculations show that, for a luminous blue variable (LBV), rotation is not expected to greatly affect the onset of dynamical instability in any direct way, but could be important through its indirect evolutionary effect on the star's luminosity-to-mass ratio. If the classical LBV's are evolving in an advanced stage of central helium burning, when their envelopes are most prone to dynamical instability, the luminosity-to-mass ratio would probably be increased by rotation. It is shown that a brightening of the star lessens its dynamical stability and so leads to a somewhat hotter effective temperature during the phase of dynamical instability. How rotation modifies the Eddington luminosity limit is also discussed.

Subject headings: stars: evolution — stars: oscillations — stars: rotation — stars: variables: other (luminous blue variables)

1. INTRODUCTION

In the conventionally presented scheme of stellar evolution, a massive star becomes a luminous blue variable (LBV) during, or shortly after, the main-sequence phase. At this time, S Doradus–type eruptions are presumed to produce an enormous loss of mass and to prevent evolution to the red (Sterken & Wolf 1978; Humphreys & Davidson 1979; Bressan et al. 1993; Meynet et al. 1994; Langer et al. 1994). However, there are problems with this picture: no demonstrated mechanism of mass loss has been discovered that actually supports the conventional scheme; the time-averaged empirical rates of mass loss from LBVs are several orders of magnitude too small to be relevant; the predicted masses, luminosities, and effective temperatures of LBVs cover much wider ranges than those actually observed; and the stellar models do not yield anything like the observed cycles of mass loss.

A more specific theory of LBVs has recently been proposed which provides many points of detailed agreement with observations over the range of luminosities where the classical LBVs are observed (Stothers & Chin 1996). The new theory is based on a demonstrated mechanism of mass loss that clearly exists in well-evolved stellar models: classical ionization-induced dynamical instability. The instability occurs in the outer layers of the envelope, during two different evolutionary phases: once, briefly, either just before the beginning or at some time during the main stages of central helium burning, when the star is a yellow or red supergiant; and later, for a much longer time, toward the end of central helium burning, when the star is again a blue supergiant with only a small hydrogen envelope left. Nearly all LBVs are predicted to be in the second phase of dynamic instability, with most of the mass loss having already taken place in or before the first phase. In essential agreement with observations are the detailed model predictions for the LBV masses, luminosities, effective temperatures, surface hydrogen abundances, lifetimes, eruptive mass-loss rates, and secular cycles of mass loss.

Although most of the evidence suggests that the basic mechanism for the LBV instability has probably been correctly identified, there do exist some discrepancies in detail. For example, the effective temperatures and surface hydrogen abundances of the models are somewhat too low. This problem is serious enough to warrant further study.

An obvious physical factor that has not been included in the models so far is axial rotation. Massive stars on the main sequence are observed to include a very large proportion of rapid rotators. Furthermore, some LBVs show nonspherical winds and surrounding nebulae, which might indicate rotation in the underlying star. It is therefore important to investigate whether the inclusion of axial rotation in the stellar models would significantly affect our current predictions. Since dynamical instability of the proposed type is governed entirely by conditions in the outer envelope, it will suffice, for the moment, to conduct a series of parameter studies of stellar outer envelopes. This can be done by simply assigning to the star a present mass, luminosity, effective temperature, surface chemical composition, and envelope rotational angular velocity.

In § 2 the usual criterion for dynamical instability is modified to include axial rotation. Applications to stellar envelope models are made in § 3, and are followed by a discussion of the main results in § 4.

2. CRITERION FOR DYNAMICAL INSTABILITY

2.1. *Exact Criterion*

Three assumptions will be made in order to test for dynamical instability in a rotating LBV. First, the rate of rotation is taken to be slow, so as to preserve, approximately, the spherical symmetry of the stellar envelope and to keep the oscillations essentially radial. (In practice, the restriction to slow rotation can be dropped without great loss of accuracy.) Second, each mass shell conserves its angular momentum during the oscillations. Third, the perturbations are assumed to be strictly adiabatic. When all of the acting forces are averaged over a spherical surface (§ 3), the linearized wave equation for small radial adiabatic displacements becomes (Stothers 1981)

$$\frac{d^2}{dr^2}\left(\frac{\delta r}{r}\right)+\left(\frac{4-V+C}{r}\right)\frac{d}{dr}\left(\frac{\delta r}{r}\right)+\frac{V}{\Gamma_1 r^2}\left[\frac{\sigma^2 r^3}{GM(r)}-\left(3\Gamma_1-4+\frac{\lambda}{1-\lambda}\right)+\frac{3\Gamma_1 C}{V}\right]\frac{\delta r}{r}=0\,. \tag{1}$$

Here δr is the displacement amplitude, σ is the adiabatic oscillation eigenfrequency (2π/period), Γ_1 is the first generalized adiabatic exponent, ρ is the density, P is the pressure, $V = -(d \ln P)/(d \ln r)$, $C = (d \ln \Gamma_1)/(d \ln r)$, $\lambda = \frac{2}{3}\Omega^2 r^3/GM(r)$, and Ω is the angular velocity of rotation. Solutions occur for those values of σ^2 for which δr is finite at the surface and zero at the base of the stellar envelope. The necessary and sufficient condition for dynamical instability is then that $\sigma^2 \leq 0$ for the lowest adiabatic mode. This adiabatic criterion is already known to be rigorously correct for the simple one-zone model of a star subjected to a fully nonadiabatic linear stability analysis (Jeans 1929; Baker 1966; Stothers 1981). There is no reason to doubt that it is applicable to a distributed stellar model as well.

Glatzel & Kiriakidis (1998) have recently contended that the *nonadiabatic* radial oscillation eigenfrequency should be used to test for dynamical instability. However, as shown by the one-zone model (Stothers & Chin 1997) as well as by numerical calculations of distributed stellar models (Tuchman, Sack, & Barkat 1978; Stothers 1999), the nonadiabatic contribution shows up only as pulsational instability, in the form of periodic oscillations superimposed on the quasi-adiabatic, dynamical expansion of the outer envelope. Dynamical instability itself is an adiabatic phenomenon, not related to the pulsational instability.

2.2. *Approximate Criterion*

For a star in slow uniform rotation, Ledoux (1945) showed analytically that rotation tends to stabilize the star dynamically. This conclusion has been verified (in great generality) by many authors, most recently by Hazlehurst (1994), who refutes some minority objections to it. Specifically, rotation increases the value of σ^2. According to Ledoux,

$$\sigma^2 = (3\langle\Gamma_1\rangle - 4)(-W/I) + (5 - 3\langle\Gamma_1\rangle)(\Omega J/I)\,, \tag{2}$$

for a star with a low central condensation. Here $\langle\Gamma_1\rangle$ is a weighted average of the first generalized adiabatic exponent, W is the total gravitational potential energy, J is the total angular momentum, and I is the total moment of inertia with respect to the center. Since the outer envelope of an LBV below the expanding atmosphere resembles an inflated balloon of negligible mass with a low central condensation (small density gradient) and the oscillation amplitude in the layers beneath the outer envelope is effectively zero (Stothers & Chin 1993; Stothers 1999), Ledoux's expression for σ^2 should apply especially well to LBVs, provided that the lower cutoff for the radius in the integral expressions for W, J, and I is applied at the base of the outer envelope rather than at the stellar center.

Ledoux (1945) gave also the conservation of energy equation:

$$W + 4\pi \int P\, d(r^3) + \Omega J = 0\,. \tag{3}$$

The criterion for dynamical instability, $\sigma^2 \leq 0$, follows from equations (2) and (3) as

$$\langle\Gamma_1\rangle \leq 4/3 - Q\,, \tag{4}$$

where

$$\langle\Gamma_1\rangle = \int_{r*}^{R} \Gamma_1 P\, d(r^3) \Big/ \int_{r*}^{R} P\, d(r^3)\,, \tag{5}$$

$$Q = \int_{r*}^{R} \frac{1}{9}\left(\frac{\lambda}{1-\lambda}\right) V P\, d(r^3) \Big/ \int_{r*}^{R} P\, d(r^3)\,, \tag{6}$$

and r^* is the radius of the base of the outer envelope.

An alternative, slightly less accurate, criterion can be derived in a similar way. By setting ρ = constant in the limit of negligible central condensation, equation (2) reduces to

$$\sigma^2 = [(5/2)(3\langle\Gamma_1\rangle - 4) + (5 - 3\langle\Gamma_1\rangle)\lambda_R]GM/R^3\,, \tag{7}$$

where λ_R is the surface value of λ. (The factor 5/2 in the first term on the right-hand side would be replaced by unity if the oscillations remained large down to the stellar center.) In the present case, the condition $\sigma^2 \leq 0$ becomes

$$\langle\Gamma_1\rangle \leq \frac{4}{3} - \frac{2\lambda_R}{3(5 - 2\lambda_R)}\,. \tag{8}$$

As a simple criterion for dynamical instability, one may adopt either equation (4) or equation (8). These two approximate criteria actually become equal and exact if the radius displacements are homologous, as they are when the equilibrium values of ρ and Γ_1 are spatially constant. The value of $\langle\Gamma_1\rangle$ is easily derived from the equilibrium properties of the stellar model, and so can be used to test for dynamical instability without having to solve the radial adiabatic wave equation (1). In the models of the present paper, however, the wave equation has been integrated exactly.

3. STELLAR MODELS

A stellar envelope, as viewed from the top down, is just an inwardly extended stellar atmosphere, and therefore it can be characterized by the star's total mass (M), luminosity (L), effective temperature (T_e), chemical abundance parameters (hydrogen fraction, X, and metallicity, Z), and envelope angular velocity (Ω).Because the gravitational acceleration is not constant deep in the envelope, M and L (rather than g) must be specified. Although our first estimates of M, L, T_e, X, and Z will be taken from the evolutionary tracks for massive stars computed by Stothers & Chin (1996), adjustments to these estimates must be made for the effects of rotation and will be discussed below. For a model of a nonrotating LBV, we choose one (called here our prototype LBV model) in a stage of marginal dynamical instability ($\sigma^2 = 0$ or $\langle\Gamma_1\rangle \approx 4/3$) taken from our evolutionary track for a star of initially 45 $M_\odot$. The model's surface parameters are $M/M_\odot = 18$, $\log (L/L_\odot) = 5.75$, $\log T_e = 4.04$, $X = 0.175$, and $Z = 0.03$. As noted earlier, T_e and X are somewhat discrepant with respect to observations; further model evolution, however, is known to bring T_e closer to the observed values while preserving the dynamical instability (Stothers & Chin 1996, Fig. 1). In all of our unstable LBV models, $\langle\Gamma_1\rangle$ is small because of the high radiation pressure relative to gas pressure and because of the extensive ionization zones of hydrogen and helium. Our particular choice of unstable model is not important here, however.

The structure of the envelope of the selected model must now be recalculated with rotation included. In order to calculate the structure of a stellar envelope in uniform rotation, even one close to breakup at the surface, the mean sphere approximation can be used and turns out to be surprisingly good (Sackmann & Anand 1970). All of the physical quantities can be understood to represent suitable averages over a spherical surface, r being a mean radius. The only changes required in the structural equations for a nonrotating stellar envelope are the multiplication of the gravitational acceleration by $1 - \lambda$ in the equation of hydrostatic equilibrium, and the multiplication of the radiative flux by $1 + 50.8\lambda^{5.27}$ in the equation of radiation transfer (Faulkner, Roxburgh, & Strittmatter 1968). The latter term can be approximated here by unity everywhere, because λ nowhere exceeds λ_R, the maximum value of which, for equatorial breakup velocity, is 0.3007 (Sackmann & Anand 1970).

Rotation affects the dynamical stability of a stellar envelope in both direct and indirect ways. The direct effects come through the introduction of the rotational kinetic energy and through the changes of the hydrostatic pressures in the envelope. Indirect effects arise from the influence of rotation on the overall evolution of the star. Thus, when the star reaches a particular evolutionary stage, the star's surface parameters (M, L, T_e, X, Z) differ from those that the star would have had in this stage if it were not rotating.

3.1. *Direct Effects of Rotation*

By reducing the effective gravity, rotation lowers the hydrostatic pressures in the envelope for a fixed effective temperature, and thus directly reduces the envelope densities. Gas pressure thereby becomes diminished relative to radiation pressure and so Γ_1 drops, an effect that tends to destabilize the envelope.

A very simple expression for the ratio of gas pressure to total pressure, β, can be derived for the case of a uniformly rotating envelope of uniform density. Integrating the equation of hydrostatic equilibrium and the equation of radiative transfer using a constant opacity, κ, and defining the radius fraction $x = r/R$, we find

$$1 - \beta = f(x)\kappa L/(4\pi cGM) , \tag{9}$$

where

$$f(x) = 2(1 - x)/(2 - 2x - \lambda_R x + \lambda_R x^3) . \tag{10}$$

If $x = 1$, $f(x) = (1 - \lambda_R)^{-1}$, whereas if $x \ll 1$, $f(x) \approx (1 - x\lambda_R/2)^{-1}$. Thus, as λ_R increases, β decreases. This effect, however, is always small, because $f(x)$ is everywhere (except near the surface) close to unity. In our prototype LBV model, $\langle\Gamma_1\rangle$ actually drops by only 0.0006 when the rotation parameter is set to its largest theoretically possible value, $\lambda_R = 0.3007$.

Nearly all of rotation's direct influence on dynamical stability, therefore, arises from the rotational kinetic energy, as seen from equation (2). Calculations based on our prototype LBV model yield for the change of the square of the nondimensional eigenfrequency $\omega^2 = \sigma^2 R^3/GM$:

$$\delta\omega^2 \approx \delta\lambda_R , \tag{11}$$

for all λ_R, in very close agreement with the approximate equation (7). Rotational kinetic energy provides a stabilizing influence.

The Eddington limit on the luminosity, with rotation included, follows from evaluating equation (9) at the mean-sphere surface with $\beta = 0$, since β cannot be negative. Thus,

$$L_E = 4\pi cG(1 - \lambda_R)M/\kappa . \tag{12}$$

Evaluated alternatively at the surface equator, R_e, this expression would have $1 - \lambda_R$ replaced by $1 - \Omega^2 R_e^3/GM$. In either case, rotation always reduces the Eddington limit. Langer (1997) approached the problem from the point of view that rotational instability occurs before the nonrotational Eddington limit is reached. Since, however, the Eddington limit was originally derived as a balance of all the forces acting (Eddington 1921), the addition of rotation to these forces simply causes a *modification* of the expression for the limiting Eddington luminosity, rather than an independent rotational instability.

3.2. *Indirect Effects of Rotation*

Rotation affects the course of stellar evolution and, as a consequence, the onset of dynamical instability. Although evolutionary tracks with rotation included have not yet been calculated for complete models of LBVs, the possible magnitudes of the changes in ω^2 are simple to compute based on our prototype LBV model envelope. It is then found that

$$\delta\omega^2 \approx 3\delta \log M , \quad \delta\omega^2 \approx -3\delta \log L , \quad \delta\omega^2 \approx -0.5\delta X . \tag{13}$$

The variations in ω^2 caused by small changes of mass and luminosity are nearly equal, although opposite in sign, because the ratio L/M is nearly constant for stellar envelopes with small β (eq. [9]). The variations arising from small changes of the hydrogen abundance X are so minor that they can evidently be ignored. However, the magnitudes (although not the signs) of the coefficients in equations (13) prove to be somewhat model dependent owing to their sensitivity to partial ionization of the gas.

4. DISCUSSION

An upper limit can be placed on the value of λ_R during the LBV phase. In the initial main-sequence stage, a star of 45 $M_\odot$ rotating uniformly at equatorial breakup velocity has $\lambda = 0.015$ in the 18 $M_\odot$ layer. Assuming local conservation of angular momentum, this layer later would show $\lambda = 0.0002$ when it eventually became exposed as the surface layer of an 18 $M_\odot$ LBV. In reality, however, there must be some redistribution of angular momentum within the evolving hydrogen envelope owing to the presence of rotational and convective mixing currents, as well as surface mass and angular momentum losses (Heger & Langer 1998). Therefore, a relatively firm upper limit for λ_R in the LBV phase in this example can be set at the value of 0.015.

With $\lambda_R \leq 0.015$, we find $\delta\omega^2 \leq 0.015$ as the direct effect of rotation on ω^2. This stabilizing influence, however, is too minor to be of much importance.

As for the indirect effects, it is already known from theoretical studies of rotating main-sequence stars that the lifting effect of the centrifugal force for an assigned value of λ_R lowers the luminosity by a small percentage that is more or less independent of the star's mass. This reduction of the luminosity lengthens the star's lifetime both during and after the main-sequence phase. In the late stages of central helium burning, however, the increase of lifetime is so pronounced that the hydrogen-burning shell adds enough new mass to the helium core as to actually increase the total luminosity of the star, at least for stars in the mass range 5–9 $M_\odot$ (Kippenhahn, Meyer-Hofmeister, & Thomas 1970; Meyer-Hofmeister 1972). This luminosity increment can amount to $\delta \log L = 0.04$ if the star was originally rotating at equatorial breakup velocity. It might be less for more massive stars, which lose a substantial amount of mass before the LBV phase and consequently have weaker hydrogen-burning shells; moreover, the luminosity rises less steeply with the helium core mass in such massive stars, and the Eddington limit on the luminosity declines with rotation in proportion to $(1 - \lambda_R)$. For a given LBV mass, therefore, $\delta \log L$ would probably be less than 0.04.

Using as a basis our published models of nonrotating LBVs with various masses (Stothers & Chin 1996), we find that the critical effective temperature at which the second phase of dynamical instability begins changes with the star's luminosity approximately as

$$\delta \log T_e \approx 4.5 \delta \log L . \tag{14}$$

This result is valid for any small luminosity increase *whether or not it is rotationally induced*. As an extreme value, we take $\delta \log L = 0.04$. The maximum shift of effective temperature is then $\delta \log T_e \approx 0.18$, although the actual rotationally induced increase is likely to be much smaller.

This upward shift, nevertheless, goes in the direction of improved agreement with the observed effective temperatures of the more luminous LBVs, which are somewhat hotter than we had predicted on the basis of nonrotating stellar models. More refined estimates of the shift would require information about the initial angular momenta and subsequent rotational histories of individual observed LBVs.

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